NCER Working Paper Series

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Working Paper #80
February 2012

Forecasting multivariate volatility in larger dimensions: some practical issues

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First version: August 2010
This version: February 2012

Abstract

The importance of covariance modelling has long been recognised in the field of portfolio management and large dimensional multivariate problems are increasingly becoming the focus of research. This paper provides a straightforward and commonsense approach toward investigating whether simpler moving average based correlation forecasting methods have equal predictive accuracy as their more complex multivariate GARCH counterparts for large dimensional problems. We find simpler forecasting techniques do provide equal (and often superior) predictive accuracy in a minimum variance sense. A portfolio allocation problem is used to compare forecasting methods. The global minimum variance portfolio and Model Confidence Set (Hansen, Lunde, and Nason (2003)) are used to compare methods, whilst portfolio weight stability and computational time are also considered.

Keywords

Volatility, multivariate GARCH, portfolio allocation

JEL Classification Numbers

C22, G11, G17

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We would like to thank Christopher Coleman-Fenn, Mark Doolan, Joanne Fuller, Stan Hurn, Andrew McClelland and Daniel Smith for their suggestions and advice. The technical support of High Performance Computing & Research Support at QUT is gratefully acknowledged. The responsibility for any errors and shortcomings in this paper remains ours.

1 Introduction

The significance of volatility modelling in a portfolio allocation sense has long been recognised. However, less is understood about the most appropriate method of handling large portfolios (that is, hundreds or thousands of assets) in this context. These large portfolios are increasingly becoming the focus of researchers and many methods have been proposed to aid in dealing with the issue of dimensionality including various estimation techniques such as composite likelihood methods proposed by Engle, Shephard, and Sheppard (2008). Extensions of Dynamic Conditional Correlation (DCC)-type models, for example equicorrelation of Engle and Kelly (2009) and blocking of Franses and Hafner (2009) and Billio, Caporin, and Gobbo (2006) also feature heavily in the literature. Surveys of the multivariate generalised autoregressive conditional heteroskedasticity (GARCH) literature include Bauwens, Laurent, and Rombouts (2006) and Silvennoinen and Teräsvirta (2009).

Recent work along the lines of this paper include Laurent, Rombouts, and Violante (2010) and Caporin and McAleer (2011). Both papers focus on the evaluation of multivariate GARCH-type models for large dimensional problems, although the latter does include an exponentially weighted moving average (EWMA) specification in its comparison of methods. Hansen, Lunde and Nason's (2003) Model Confidence Set (MCS) methodology is implemented in both as an evaluation tool and the out-of-sample periods are divided into subsamples based on the relative level of volatility. This is also the case in our paper. The benefits of our choice of the MCS as an evaluation tool are twofold: it does not require a benchmark model to be specified; and is a statistical test of the equivalence of a given set of forecasting methods with respect to a particular loss function. In our case, the loss function will be the variance of returns from the global minimum variance portfolios computed from each of the forecasts. In terms of the out-of-sample period, investigation of the forecasting performance of models under differing volatility conditions is becoming more popular in the literature (see Luciani and Veredas (2011) for a recent example) and is of great interest here given the market turbulence of recent years.

This paper differs from Laurent, Rombouts, and Violante (2010) and Caporin and McAleer (2011) in two important ways: firstly, the use of daily data as opposed to intraday allows scope for larger dimensional portfolios (the largest number of assets, N, here is $N = 200$ compared to $N = 89$ in the daily data application in Caporin and McAleer (2011)). Daily data allows us to circumvent a number of difficulties posed by the use of high frequency data for large dimensional problems (big N), for example limitations imposed by stock liquidity. Notably, for $N > T$ the positive definitiveness of the covariance matrix becomes a problem. Secondly, a wider range of 'simple' methods are considered here, shifting the focus to a more practical, less GARCH-orientated study. This paper investigates and evaluates popular methods for forecasting large covariance matrices and specifically considers whether equal, or perhaps superior forecasts of the conditional covariance matrix can be achieved using much simpler means than more complex and often computationally cumbersome methodologies.

An empirical portfolio allocation exercise is used to compare various covariance forecasting techniques. Minimum variance portfolios are formed and the out-of-sample performance of the methods compared, using the MCS of Hansen, Lunde, and Nason (2003). In addition to the question of minimising the volatility of the portfolio, more practical aspects including the stability of the resulting portfolio and the central processing unit (CPU) time of the forecasting methods are considered. The various models are compared across a number of portfolio sizes and for both a full out-of-sample application as well as periods of relatively high and low levels of market volatility.

The paper proceeds as follows. Section 2 details the dataset used here. Section 3 outlines the forecasting methods compared and how they will be implemented. Section 4 considers the tools used for comparison and reports the results of the empirical application. Section 5 concludes.

2 Data

The portfolios used contain a selection of S&P1500 stocks that continuously traded over the period 03/01/1994 to 31/12/2009. The full dataset contains 200 stocks ($N = 200$) and 4029 observations ($T = 4029$). All GICS sectors are represented across the dataset and the full list of stocks including their ticker code, company name and sector is provided in the Appendix. Over 60% of the assets contained in the dataset represent the Information Technology, Industrials, Financials and Consumer Discretionary sectors. Log returns, $r_{i,t}$, are calculated using $[r_{i,t} = \log p_{i,t} - \log p_{i,t-1}]$ where $p_{i,t}$ denotes the daily closing price of asset i at time t.

The in-sample period is 2000 observations, allowing for 2029 one-step-ahead forecasts. Descriptive statistics for the in-sample, out-of-sample and total sample periods are provided in Table 1 for the stocks with the highest and lowest volatility for the given period as measured by the unconditional standard deviation. For further comparison, descriptive statistics for the S&P1500 composite index daily returns are also provided, beginning 02/11/94. The S&P500 index covering the entire sample period yielded very similar statistics and thus not included here.

Of note are the periods of relative high and low volatility over the sample. The upper panel of Figure 1 shows the S&P1500 returns series and the lower panel the squared returns series. The beginning of the sample is characterized by relatively low volatility, followed by a higher overall level of volatility. This high volatility spans a period from around April 1997 until July 2003. The following 3 or so years are again a time of lower overall market volatility. Finally, the last portion of the sample (from around March 2007) is one of higher overall volatility. This period corresponds to the recent global financial crises (GFC). These overall changes are of interest as we look into any possible effect the overall level of volatility has on the relative performance of the forecasting methods. It is becoming increasingly common for researchers to evaluate forecasting methods for sub-periods of differing levels of volatility, see Luciani and Veredas (2011) for a recent example.

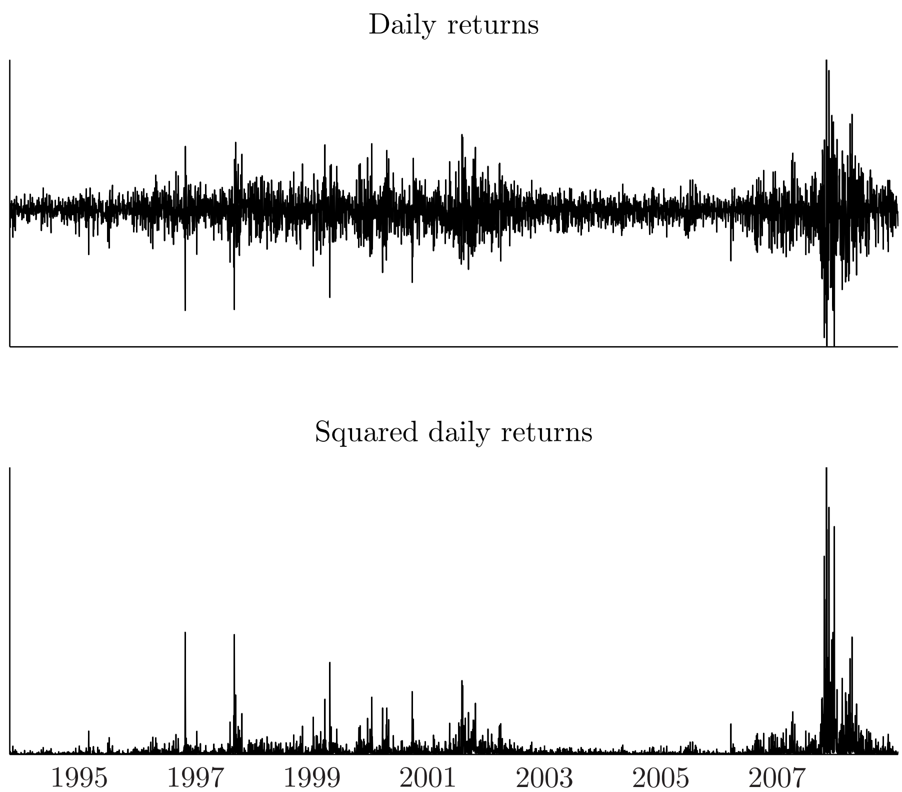

Figure 1: Daily returns, r_t, of the S&P1500 index (*Upper Panel*) and squared daily returns (*Lower Panel*). Period spanning 03/01/1994 - 31/12/2009.

		Symbol	Min	Max	$\bar{x}$	s	Skewness	Kurtosis
In-sample	Camden Property Trust	CPT	-0.0750	0.0537	5.0582	16.8285	-0.1375	6.2715
	Comtech Tele. Corp	CMTL	-0.2077	0.2803	14.4479	90.3525	0.4010	4.6894
	S&P1500 Index	SPR	-0.0305	0.0213	5.4333	7.5656	-0.3295	6.9145
Out-of-sample	The Clorox Co.	CLX	-0.0843	0.0943	5.4225	21.2299	-0.0035	8.2595
	CryoLife Inc.	CRY	-1.0003	0.7744	-18.3879	86.1465	-1.9321	88.9126
	S&P1500 Index	SPR	-0.0417	0.0468	0.1383	9.7101	-0.1648	11.7706
Total	AGL Resources Inc.	AGL	-0.1037	0.0822	4.0169	21.6675	-0.0403	7.8881
	CryoLife Inc.	CRY	-1.0003	0.7744	7.0422	78.0060	-1.3242	68.1387
	S&P1500 Index	SPR	-0.0417	0.0468	2.6201	8.7709	-0.2315	11.1850

Table 1: Summary statistics of the lowest and highest volatility stocks for the in-sample, out-of-sample and total sample periods as measured by the unconditional standard deviation, s. Sample mean of a given time period is denoted $\bar{x}$. In-sample period of 2000, entire period spanning 03/01/94 - 31/12/09. S&P1500 sample begins 02/11/94.

3 Generating and evaluating forecasts

A common decomposition of the covariance matrix (see Engle (2002)) is

$$\mathbf{H}_{t+1} = \mathbf{D}_{t+1}\mathbf{R}_{t+1}\mathbf{D}_{t+1} \tag{1}$$

where $\mathbf{H}_{t+1}$ is the covariance matrix at time $t+1$, $\mathbf{R}_{t+1}$ the correlation matrix at time $t+1$ and D_{t+1} the diagonal matrix of univariate standard deviations at time $t+1$. In order to compute the elements of $\mathbf{D}_{t+1}$, the $d_{i,t+1}$'s, where i denotes the asset, univariate volatility modelling is used. Consider here the asset return series, $r_{i,t}$, shown in (2): it is the product of the conditional standard deviation or $\sigma_{i,t}$ and the standardised disturbance term $\epsilon_{i,t}$, where $\epsilon_{i,t}$ is normally distributed with mean 0 and variance 1.

$$r_{i,t} = \sigma_{i,t}\, \epsilon_{i,t}, \qquad i = 1,\ 2,\ \dots,\ n \tag{2}$$

Engle and Patton (2001) outline a number of stylised facts regarding volatility, the first (and arguably most important) of these is that volatility exhibits persistence; that is, the clustering of volatility shocks so that a large move (of either sign) will be followed by another large move and so forth. It can be shown that $r^2_{i,t} = \sigma^2_{i,t}$, where the squared return is considered an unbiased proxy of the volatility. Although noisy, squared returns are adequate here, given 'simple' approaches to covariance modelling are of interest. This concept of volatility persistence coupled with the specification of $r_{i,t}$ in (2) is the basis of the empirically successful autoregressive conditional heteroskedasticity (ARCH) family of model, first proposed by Engle (1982). The ARCH model allows the conditional variance to vary over time dependent on the past squared forecast errors.

The most commonly applied extension of ARCH is the Generalised ARCH (GARCH) model of Bollerslev (1986), a successful predictor of conditional variances even in its simplest form. The GARCH (p, q) model (3) is mean reverting and conditionally heteroskedastic with a constant unconditional variance.

$$h_{i,t} = \alpha_{i,0} + \sum_{j=1}^{q} \alpha_{i,j} r^2_{i,t-j} + \sum_{j=1}^{p} \beta_{i,j} h_{i,t-j} \tag{3}$$

where $h_{i,t}$ is the univariate variance of asset i at time t and $h_{i,t-j}$ the j-th lag; $r^2_{i,t-j}$ the squared return of asset i at time $t-j$; and $\alpha_{i,0}$, $\alpha_{i,j}$ and $\beta_{i,j}$ parameters constrained to $\alpha_{i,0} \geq 0$, $\alpha_{i,j} \geq 0$, $\beta_{i,j} \geq 0$ and $\alpha_j + \beta_j < 1$ for the i-th asset.

Along with exhibiting persistence, volatility reacts asymmetrically to past forecast errors such that in a financial sense, negative returns seem to have a larger influence on future volatility than positive ones. Nelson's (1991) Exponential GARCH sought to address this characteristic of volatility by incorporating the sign of a return, rather than its magnitude alone. Similarly, the model of Glosten, Jagannathan, and Runkle (1993) (GJR-GARCH) addresses asymmetry in volatility, by including a dummy variable that takes the value 1 should the asset return be negative. The specification of such a model is provided in (4), where $I_{i,t-j}$ is the indicator variable for asset i at time $t-j$ and $\delta_{i,j}$ the relevant parameter. The constraints of the previous model now become $\alpha_{i,0} \geq 0$, $\alpha_{i,j} + (\delta_{i,j}/2) \geq 0$, $\beta_{i,j} \geq 0$ and $\alpha_{i,j} + (\delta_{i,j}/2) + \beta_{i,j} < 1$.

$$h_{i,t} = \alpha_{i,0} + \sum_{j=1}^{q} \alpha_{i,j} r_{i,t-j}^2 + \sum_{j=1}^{q} \delta_{i,j} r_{i,t-j}^2 I_{i,t-j} + \sum_{j=1}^{p} \beta_{i,j} h_{i,t-j} \tag{4}$$

Consistent with the multivariate GARCH literature, the models are estimated using a two-stage procedure whereby each series of returns, $\boldsymbol{r}_{i,t}$, is standardised using a univariate GARCH-type model. As is usually the case, the volatility-standardised returns will be denoted as $\boldsymbol{\epsilon}_{i,t} = \boldsymbol{r}_{i,t}/\sqrt{\boldsymbol{h}_{i,t}}$. The underlying volatility process here is the GJR-GARCH(1, δ, 1) of Glosten, Jagannathan, and Runkle (1993) and the original GARCH(1, 1) of Bollerslev (1986). To avoid any potential cost associated with estimating the asymmetry parameter δ unnecessarily (Thorp and Milunovich (2007)), the significance of this parameter was tested. Thirteen stocks were found to have insignificant (at the 5% level) δ's and thus their volatility processes are estimated using GARCH(1, 1). The remaining 187 stocks' univariate volatility processes are estimated using GJR-GARCH(1, δ, 1). The volatility-standardised returns are used to generate one-step-ahead correlation matrices, $\mathbf{R}_{t+1}$, for each of the competing models are scaled by the univariate standard deviations, $\mathbf{D}_{t+1}$, to equal the conditional covariance matrix, $\mathbf{H}_{t+1}$, in (1). The forecasts of the conditional covariance matrix are used to determine the optimal portfolio weights for each forecasting method.

Increasingly over recent years, researchers have been interested in portfolio allocation applications of volatility timing that are not limited in terms of the number of assets modelled. Previously, work was limited in scope: primarily due to the problems associated with the estimation of multivariate GARCH (M-GARCH) models with time-varying correlations. The two well publicized complications in developing a conditional covariances forecasting tool are (a) the statistical requirement that the covariance matrix be positive definite; and (b) that the model be effectively parsimonious to avoid parameter proliferation when modelling the conditional covariance of multiple time series. As in the univariate literature, the multivariate work is wide ranging. Surveys of the M-GARCH literature include Bauwens, Laurent, and Rombouts (2006) and the more recent work of Silvennoinen and Teräsvirta (2009).

The Dynamic Conditional Correlation (DCC) model of Engle and Sheppard (2001), formally introduced by Engle (2002), is considered to be a parsimonious approach addressing both issues. A generalisation of Bollerslev's (1990) Constant Conditional Correlation (CCC) model, the DCC framework firstly estimates univariate GARCH models for each series. Utilising the standard residuals obtained in the first instance, a so-called 'pseudo' time varying correlation matrix is then estimated using the $\mathbf{Q}_{t+1}$ specification[2] in (5).

$$
\begin{aligned}
\mathbf{H}_{t+1} &= \mathbf{D}_{t+1}\mathbf{R}_{t+1}\mathbf{D}_{t+1} \\
\mathbf{R}_{t+1} &= \mathrm{diag}(\mathbf{Q}_{\mathrm{t+1}})^{-1/2}\mathbf{Q}_{\mathrm{t+1}}\mathrm{diag}(\mathbf{Q}_{\mathrm{t+1}})^{-1/2} \\
\mathbf{Q}_{t+1} &= \bar{\mathbf{Q}}(1-\alpha-\beta) + \alpha\ \mathrm{diag}(\mathbf{Q}_t)^{1/2}\boldsymbol{\epsilon}_{t-1}\boldsymbol{\epsilon}'_{t-1}\ \mathrm{diag}(\mathbf{Q}_t)^{1/2} + \beta\ \mathbf{Q}_{t-1}
\end{aligned}
\tag{5}
$$

where $\mathbf{D}_{t+1}$ is a diagonal matrix of conditional standard deviations, computed from the univariate volatility models underlying the DCC; α and β are parameters subject to the constraints $\alpha > 0$, $\beta > 0$ and $\alpha+\beta < 1$; and $\boldsymbol{\epsilon}_{i,t} = \boldsymbol{r}_{i,t}/\sqrt{\boldsymbol{h}_{i,t}}$ the volatility-standardised returns. Persistence in returns is demonstrated should the sum of the two parameters be close to unity, implying that the closer the sum is to one the more persistent the correlations. As the parameters here are scalar values, the correlation dynamics are equal for all assets.

In terms of estimation, the DCC is often estimated using a two-stage quasi-maximum likelihood procedure. The second stage of the log-likelihood is given in (6) and is included here to allude to a potential issue for this type of estimator in the large dimensional scenario. In particular, the requirement of inverting the correlation matrix $\mathbf{R}_t$. For standard maximum likelihood optimisation routines, this term will be computed for each t a number of times. For large N, inversion of this matrix becomes numerically intensive. This point is both important and

[2] This is the specification offered by Aielli (2009). The literature points to the use of this model in place of the original.

relevant to the practical implementation of any empirical application of this model.

$$\log L = -\frac{1}{2}\sum_{t=1}^{T}\left(n\log(2\pi) + 2\log(|\mathbf{D}_t|) + \log(|\mathbf{R}_t|) + \boldsymbol{\epsilon}_t'\mathbf{R}_t^{-1}\boldsymbol{\epsilon}_t\right) \tag{6}$$

To address the problem of bias afflicting the two-step quasi-likelihood estimators, Engle, Shephard, and Sheppard (2008) introduced a composite likelihood approach to covariance modelling and thus rendered these methods plausible for large-scale applications. The composite likelihood is constructed and subsequently maximised to provide the estimate of the covariance matrix. This approach has been used successfully in the mathematics literature for some time (see Lindsay (1988) and more recently, Varin and Vidoni (2005)) for applications where standard likelihood methods are infeasible.

The composite likelihood is the sum of quasi-likelihoods, obtained by breaking the portfolio of assets into subsets. Each subset will yield a quasi-likelihood estimator, which can then be added to the others to produce the composite likelihood. The process avoids having to invert large covariance matrices, preventing burdensome computational issues and also the bias introduced by an unknown incidental parameter. The estimator can be $O(1)$ if necessary and will remain unbiased even if the number of assets exceeds that of the observations (or rather, the number of time steps).

Formally, consider the K-dimensional vector of log-returns, $\mathbf{r}_t$, where $t = 1, 2, ..., T$. The covariance matrix $\mathbf{H}_t$ is modelled using past data by estimating a number of parameters. As alluded to above, the portfolio is split into subsets: effectively transforming a vast dimensional system into a number of small ones. To do so, $\mathbf{r}_t$ is transferred into the data array $\mathbf{Y}_t = \{Y_{1,t}, ..., Y_{N,t}\}$ where $\mathbf{Y}_{j,t}$ is a vector of small subsets of the data. This can be shown as $\mathbf{Y}_{j,t} = \mathbf{S}_j\ \mathbf{r}_t$, where $\mathbf{S}_j$ is a non-stochastic selection matrix. In their paper, Engle, Shephard, and Sheppard (2008) consider all unique pairs of data where $N = K(K-1)/2$ and this is the approach used here. A valid quasi-likelihood for the j-th subset is constructed to estimate the parameters. By averaging over a number of submodels and summing over the series a sample composite likelihood (CL hereafter in accordance with the literature) function is produced. Evaluation of the CL costs $O(K^2)$ calculations, gaining an advantage over standard quasi-likelihood methods.

In addition to using the CL approach to estimating and subsequently generating covariance forecasts using the DCC model, this paper is interested in the application of much simpler methods to forecast covariances for multivariate systems. The concept of univariate volatility persistence also implies tomorrow's covariance is dependent on that seen historically, and so the simplest forecasting tool is a simple moving average as in (7).

$$\mathbf{Q}_t^{SMA} = \frac{1}{K}\sum_{j=1}^{K} \boldsymbol{\epsilon}_{t-j}\boldsymbol{\epsilon}'_{t-j} \tag{7}$$

where K is the moving average period (referred to as the 'rolling window'), $\boldsymbol{\epsilon}_{t-j}\boldsymbol{\epsilon}'_{t-j}$ the jth lag of the cross product of the volatility standardised returns' series, and $\mathbf{Q}_t^{SMA}$ the forecasted pseudo-correlation matrix. Note here that the cross product of the standardised returns series, $\boldsymbol{\epsilon}_{t-j}\boldsymbol{\epsilon}'_{t-j}$, is used as a predictor of the correlation matrix, denoted by $\mathbf{Q}_t^{SMA}$. This matrix is not quite in the form of a correlation matrix (hence the term 'pseudo-correlation'), so is rescaled using the same method as a DCC-type model (the general case is shown in (8)). Further, as long as $N < K$ the symmetric covariance matrix that results should be positive definite (Chiriac and Voev (2011)). To ensure this is the case, a 252-day rolling window is used (this corresponds to a trading year). The use of a full trading year is also consistent with Value at Risk (VaR) applications, in accordance with the Basel Committee on Banking Supervision (1996).

$$\mathbf{R}_t = \text{diag}(\mathbf{Q}_\text{t})^{-1/2}\mathbf{Q}_\text{t}\text{diag}(\mathbf{Q}_\text{t})^{-1/2} \tag{8}$$

Of course, the forecasted covariance matrix $\mathbf{H}_{t+1}$ is of primary interest here and is found using the equation shown in (1). The simple moving average is a popular tool of technical traders and investors in both a univariate and multivariate setting due to its practical and computationally quick (relative to the other models discussed) application.

The natural extension of this basic model is the exponentially-weighted moving average which places a higher emphasis on more recent observations. J.P. Morgan and Reuters's (1996) Risk-Metrics examine the exponential filter in detail and Fleming, Kirby, and Ostdiek (2001) extend their specification to that in (9).

$$\mathbf{Q}_t^{EWMA} = \exp(-\alpha)\mathbf{Q}_{t-1}^{EWMA} + \alpha\exp(-\alpha)\boldsymbol{\epsilon}_{t-1}\boldsymbol{\epsilon}'_{t-1} \tag{9}$$

The rate of decay, $\exp(-\alpha)$ can be computed using standard optimisation routines. Rather than estimating it here, a 252-day window is used and the decay rate α is set using $\alpha = 2/(K+1)$ (Morgan and Reuters (1996)). Again, the forecasted pseudo-correlation matrix is denoted as

$\mathbf{Q}_t^{EWMA}$ and thus must be rescaled using (8).

Fleming, Kirby, and Ostdiek's 2001 and 2003 papers examine the potential gain of volatility timing using the exponential weighting scheme in (9). The reasoning behind this is straightforward, that is if $\mathbf{Q}_t^{EWMA}$ is indeed time varying, the covariance dynamics will be reflected in the path of the returns. Thus, employing a method that requires the squares and cross-products of the lagged returns is ideal. Their choice of an exponential estimator is also well founded, as Foster and Nelson (1996) show the scheme will in general provide the smallest mean squared error (MSE). In addition, positive definiteness of the resulting conditional covariance matrix is assured.

Both moving average techniques outlined here are simplistic in nature and are commonly thought of as a 'simple' class of covariance forecasting tools. They require little if any optimisation at all and can be thought of as non-parametric (in the case of (9), this requires the parameter α to be fixed as is the case here). The third method used is the MIxed DAta Sampling (MIDAS) model of of Ghysels, Santa-Clara, and Valkanov (2006). It does not require maximum likelihood optimisation despite being parametric in nature and is not classed as a GARCH-type model, thus is considered another 'simple' estimator in this paper.

The MIDAS approach is shown in (10).

$$\mathbf{Q}_t^{MIDAS} = \bar{\mathbf{Q}} + \phi \sum_{k=0}^{k^{max}} \theta_k \boldsymbol{\epsilon}_{t-k} \boldsymbol{\epsilon}'_{t-k} \tag{10}$$

where $\mathbf{Q}_t^{MIDAS}$ is the forecasted pseudo-correlation; $\bar{\mathbf{Q}}$ the mean or rather, unconditional sample correlation; scale parameter ϕ; the polynomial lag parameters, or weights, θ_k; the maximum lag length k^{max}; and the forecasting variable $\boldsymbol{\epsilon}_{t-k}\boldsymbol{\epsilon}'_{t-k}$ as used above.

In keeping with Ghysels, Santa-Clara, and Valkanov (2006), the weighting scheme here will be based on the Beta function, so that $\theta = [\theta_1, \theta_2]$. For the purposes of volatility forecasting θ_1 can be restricted to equal 1 and $\theta_2 < 1$ implying a slow decay typical of such models (and ensuring only one parameter need be estimated). The resulting weights are normalised to sum to one, allowing estimation of the scale parameter ϕ_H. As in the case of the EWMA model, no estimation is undertaken. Rather, the parameter θ_2 is set to be 0.98 implying slow decay and k^{max} to be 252 days (consistent with the previous moving averages). The MIDAS framework is becoming increasingly popular for a range of applications, however most focus on univariate implementation of the model. To the best of my knowledge, the use of the MIDAS specification to estimate and forecast the conditional covariances of a portfolio of assets (of size greater than 2) remains a largely open area in the literature.

The final forecasting approach used here is Dynamic Equicorrelation (DECO) (Engle and Kelly (2009)), the correlation matrix $\mathbf{R}_{t+1}$ provided in (11) is the point of difference between this model and the DCC method described above.

$$\mathbf{R}_{t+1} = (1 - \rho_{t+1})\mathbf{I}_N + \rho_{t+1}\mathbf{1} \tag{11}$$

where ρ_{t+1} is the equicorrelation at time $t+1$; $\mathbf{I}_N$ the N-dimensional identity matrix; and $\mathbf{1}$ a $N \times N$ matrix of ones. All pairs of returns are restricted to have equal correlation on a given day.

Engle and Kelly (2009) assert that by specifying a conditional volatility model and a correlation process, the DECO framework can be applied to individual problems with success. The example provided involves use of a DCC-type approach to specify the correlations then averages the pairwise correlations to determine the parameters. Thus the $\mathbf{Q}_{t+1}$ matrix of standardised returns in this case is:

$$\mathbf{Q}_t = \bar{\mathbf{Q}}(1 - \alpha - \beta) + \alpha \ \mathrm{diag}\mathbf{Q}_t^{1/2} \boldsymbol{\epsilon}_t \boldsymbol{\epsilon}_t' \ \mathrm{diag}\mathbf{Q}_t^{1/2} + \beta \ \mathbf{Q}_t \tag{12}$$

DECO averages the pairwise DCC correlations to obtain ρ_{t+1}. Thus,

$$\rho_{t+1} = \frac{1}{n(n-1)} \left(\mathbf{1}' \mathbf{R}_{t+1}^{DCC} \mathbf{1} - n \right) = \frac{2}{n(n-1)} \sum_{i>j} \frac{q_{i,j,t+1}}{\sqrt{q_{i,i,t+1} q_{j,j,t+1}}} \tag{13}$$

where $q_{i,j,t+1}$ is the i,j th element of $\mathbf{Q}_{t+1}$. To estimate the model, univariate GARCH models are computed in the first stage as per the DCC quasi-maximum likelihood approach and the correlation matrix $\mathbf{R}_{t+1}$ in (11) is used. The difference between the equicorrelation method and standard DCC is best shown be comparing the second step of the DECO likelihood to that of the DCC (6).

$$L = -\frac{1}{T} \sum_t \left[\log \left([1 - \rho_t]^{n-1} [1 + (n-1)\rho_t] \right) + \frac{1}{1 - \rho_t} \left(\sum_i (\epsilon_{i,t}^2) - \frac{\rho_t}{1 + (n-1)\rho_t} \left(\sum_i \epsilon_{i,t} \right)^2 \right) \right]$$

where $\boldsymbol{\epsilon}_t$ are the returns standardised for the first-stage volatility estimates, $\boldsymbol{\epsilon}_t = \mathbf{D}_t(\hat{\theta})^{-1} \boldsymbol{r}_t$ and ρ_t given by (13). This approach avoids the inversion of the $\mathbf{R}_t$ matrix and so is less burdensome and computationally quicker to estimate than the DCC framework.

Global minimum variance portfolio weights, variances and the Model Confidence Set (MCS) of Hansen, Lunde, and Nason (2003) are compared in order to evaluate forecasting performance of the competing models. Evaluation of the forecasts centers on generation of global minimum variance portfolios, with weights $\boldsymbol{w}_t$ as shown in (15).

$$\boldsymbol{w}_t^{*T}\mathbf{H}\,\boldsymbol{w}_t^{*} = \min_{\boldsymbol{w}_t^{*}\in\,\mathbb{R}^N} \boldsymbol{w}_t^{T}\mathbf{H}\,w \tag{14}$$

$$\boldsymbol{w}_t^{*} = \frac{\mathbf{H}^{-1}\mathbf{1}}{\mathbf{1}^T\mathbf{H}^{-1}\mathbf{1}} \tag{15}$$

The benefits of utilising the global minimum variance portfolio as the loss function for this problem center on not needing to specify or make assumptions regarding the expected return of the portfolio. Both Caporin and McAleer (2011) and Clements, Doolan, Hurn, and Becker (2011) employ the global minimum variance portfolio as a useful comparison criteria of various covariance forecasts. An equally-weighted portfolio is also generated as a useful benchmark to which all models can be compared.

The Model Confidence Set (MCS) proposed by Hansen, Lunde, and Nason (2003), is used to evaluate the significance of any differences in performance between models. Papers using the MCS in similar multivariate settings include Clements, Doolan, Hurn, and Becker (2009) and Laurent, Rombouts, and Violante (2010), among others. The premise of the MCS procedure is to avoid specifying a benchmark model, rather it begins with a full set of candidate models $\mathcal{M}_0 = 1, ..., m_0$ and sequentially discards elements of $\mathcal{M}_0$ to achieve a smaller set of models. This model confidence set will contain the best model with a given level of confidence $(1-\alpha)$.

To find the MCS, we will follow Clements, Doolan, Hurn, and Becker (2009). The loss function is defined as

$$\mathcal{L}(H_t) = w_t^T r_t r_t^T w_t \tag{16}$$

and specifying the loss deferential between models i and j at time t as

$$d_{ij,t} = \mathcal{L}(H_t^i) - \mathcal{L}(H_t^j)\ , \qquad\qquad i,j = 1, ..., m_0\ . \tag{17}$$

The procedure involves testing the following

$$H_0 \ : \ \mathbb{E}(d_{ij,t}) = 0 \ , \qquad\qquad \forall \ i > j \in \mathcal{M} \tag{18}$$

for a set of models $\mathcal{M} \subset \mathcal{M}_0$. The initial step sets $\mathcal{M} = \mathcal{M}_0$. The model with the worst performance is removed from the set if the null is rejected at significance level α and the test performed again. The process continues until failure to reject the null and the resulting set of models is the MCS, denoted $\widehat{\mathcal{M}}*_\alpha$. The t-statistic, t_{ij} in (19), scales the average loss differential of models i and j by $\widehat{\text{var}}(\bar{d}_{ij})$. The estimate of the variance of average loss differential can be obtained using the bootstrap procedure in Hansen, Lunde, and Nason (2003).

$$t_{ij} = \frac{\bar{d}_{ij}}{\sqrt{\widehat{\text{var}}(\bar{d}_{ij})}} \ , \qquad\qquad \bar{d}_{ij} = \frac{1}{T}\sum_{t=1}^{T} d_{ij,t} \tag{19}$$

These $(m-1)m/2$ t-statistics are converted into one test statistic using (20), referred to as the range statistic, with rejection of the null hypothesis occurring for large values of the statistic. The worst performing model, determined by (21), is removed from $\mathcal{M}$ and the entire procedure repeated on the new, smaller set of models. Iterations continue until the null hypothesis is not rejected, the resulting set of models is the MCS.

$$T_R = \max_{i,j \in \mathcal{M}} |t_{ij}| = \max_{i,j \in \mathcal{M}} \frac{|\bar{d}_{ij}|}{\sqrt{\widehat{\text{var}}(\bar{d}_{ij})}} \tag{20}$$

$$i = \arg \max_{i \in \mathcal{M}} \frac{\bar{d}_{ij}}{\sqrt{\widehat{\text{var}}(\bar{d}_{ij})}} \ , \qquad\qquad \bar{d}_{ij} = \frac{1}{m-1}\sum_{j \in \mathcal{M}} \bar{d}_{ij,t} \tag{21}$$

4 Empirical Results

An initial in-sample period of 2000 observations is used, giving an out-of-sample period of $T_{fc} = 2029$. Recursive estimation is employed where appropriate, and the forecasting horizon will be one day. To further assess any differences in practical application of a number of the methods, forecast-only versions of both DCC-CL and DECO are computed (denoted DCC-EX and DECO-EX). That is, the respective correlation parameters are computed once for the in-sample period (2000 observations) and the resulting parameter estimates used to forecast the one-step-ahead covariance matrix for the entire out-of-sample period, that is no recursive estimation. Portfolios used here contain $N = [5, 10, 50, 100, 200]$ assets, randomly chosen from the list of 200 stocks of the S&P1500 as described in Section 2 (list available in the Appendix).

Results presented in Table 2 are the out-of-sample standard deviations (s) of each of these portfolios, across the various portfolio sizes and models. The equally-weighted portfolio is beaten by all models in all cases, as is expected. So do 'simple' forecasting methods result in forecasts of equal predictive accuracy compared to more complex specifications (such as the M-GARCH family)? Overall a number of the simple approaches lead to similar outcomes for the global minimum variance portfolio (GMVP). For the various portfolio sizes, the MIDAS, Fleming-Kirby-Ostdiek 252-day version of the EWMA (EWMA (FKO)) and 252-day simple moving average (SMA) perform well relative to the M-GARCH models. For the smaller value of $N = 5$ there is little difference between models, with the exception of the equally-weighted portfolio. The DCC-CL method is outperformed by DECO for moderate and large values of N, that is 10 to 200 assets, and both are outperformed by the 'simple' models. The magnitude of this out-performance seems to increase as N does. It is thought that DECO has less estimation error relative to the DCC-CL model as N increases. In terms of the DCC-CL method, as portfolio size increases, the estimation error dominates due to the necessary estimation of the unconditional correlation matrix (see Ledoit and Wolf (2004) for discussion of estimation error and the sample covariance matrix). Equicorrelation has previously been found to be useful as a shrinkage target by Ledoit and Wolf (2004) and the usefulness of the assumption of equicorrelation is also apparent here in the portfolio allocation context. In the cases where $N = 100$ and $N = 200$ (that is, N is large), there appears to be no significant difference between the simpler methods themselves, suggesting persistence in correlations (although important to allocation) may be incorporated into the covariance forecast by using a simple moving average. As an aside, there appears to be no advantage to daily re-estimation of the DCC-CL and DECO correlation parameters, as there is very little (if any) difference between the out-of-sample standard deviations of these methods with their respective forecast-only version (denoted DCC-EX and DECO-EX). For the forecast-only versions, the correlation parameters were estimated once using the in-sample period and forecasts generated with no subsequent estimation of parameters[3].

[3] DECO can be used to obtain better estimates of the DCC correlation parameters (see Engle and Kelly (2009)), however the results obtained here show that once-off estimation of these parameters is adequate.

	EQ-W	MIDAS (0.98)	SMA	EWMA (FKO)	DCC-CL	DECO	DCC-EX	DECO-EX
N	s	s	s	s	s	s	s	s
5	22.1580	20.8938	20.8918	20.8526	20.9228	20.9859	20.9229	20.9772
10	19.9630	16.4263	16.9930	16.2930	17.7450	17.6072	17.7461	17.5910
50	21.3140	12.2997	12.4112	12.0866	17.4217	15.0600	17.4217	14.9720
100	19.8750	12.6359	13.0940	12.6518	16.4830	15.7187	16.4740	15.7797
200	19.4574	14.6116	14.1757	14.2679	16.4741	15.8257	16.4740	15.7749

Table 2: Out-of-sample summary statistics of minimum-variance portfolio returns for each volatility timing strategy: s is the annualised percentage volatility of portfolio returns. In-sample period of 2000, entire period spanning 03/01/94 - 31/12/09.

N	EQ-W	MIDAS (0.98)	SMA	EWMA (FKO)	DCC-CL	DECO	DCC-EX	DECO-EX
5	0.0040	0.4540*	0.6510*	1.0000*	0.4540*	0.0040	0.4540*	0.0040
10	0.0060	0.0130	0.0070	1.0000*	0.0070	0.0060	0.0070	0.0060
50	0.0000	0.0060	0.0060	1.0000*	0.0000	0.0000	0.0000	0.0000
100	0.0010	1.0000*	0.3500*	0.9570*	0.0010	0.0010	0.0560*	0.0010
200	0.0040	0.7600*	1.0000*	0.9440*	0.0550*	0.0550*	0.1610*	0.0550*

Table 3: Empirical MCS of out-of-sample global minimum-variance portfolio. Range MCS p-values are used; * indicates the model is included in the MCS with 95% confidence.

Table 3 contains the MCS p-values for the out-of-sample global minimum-variance portfolios for each forecasting approach. Unsurprisingly, the equally-weighted portfolio is removed from the MCS for all values of N. DECO is also omitted from the MCS for the small $N = 5$ and moderate ($N = 10$ and $N = 50$) portfolio sizes. All other methods are included in the MCS for $N = 5$. For moderate portfolio sizes of 10 and 50 stocks, the only covariance forecasting model in the MCS is the Fleming-Kiby-Ostdiek 252-day version of the EWMA (EWMA (FKO)). This suggests that for moderate values of N, a simple method is the most appropriate over the given sample period. The performance of the EWMA (FKO) model across the moderate portfolio sizes are consistent with those found in Clements, Doolan, Hurn, and Becker (2011). For larger values of N (100 and 200 assets), the MCS contains a wider range of models, with the unequal weighting scheme of the MIDAS and EWMA (FKO) models considered the superior choices across the various models included in the MCS. Both M-GARCH models are included for the largest portfolio of 200 assets. In keeping with the analysis of Table 2, these results suggest that simpler methods are indeed appropriate for modelling the covariance of larger portfolios. Also in line with the results of Table 2 is the performance of the forecast-only versions of the M-GARCH models (DCC-EX and DECO-EX) being very similar to the daily re-estimation version, with the exception of the 100 stock portfolio where DCC-EX is included in the MCS and its daily estimation counterpart is not. This suggests that from a practical point of view re-estimation of parameters is unnecessary, a useful saving of computation time. This point is elaborated on in terms of CPU time later in this section.

To further analyze these findings, the out-of-sample period has been split into periods of relatively 'high' and 'low' volatility. In this dataset, the periods of higher volatility correspond to the market turbulence of 2001-2002 and the recent global financial crisis (GFC), beginning toward the end of 2008 through to the end of the dataset. The annualised percentage volatilities of global minimum variance portfolio returns, s, in Table 4 show similar patterns to Table 2. The equally-weighted portfolio is beaten in all cases. Overall, simple methods appear to result in smaller portfolio variances relative to the M-GARCH methods when the out-of-sample period is split into subsamples of high and low volatility periods. In addition, some differences are seen between the two high volatility periods in terms of the model that generates the portfolio with the smallest variance. This is perhaps due to the overall higher levels of volatility seen during the global financial crisis in comparison to the 2001-2002 turbulence. For the first high volatility period the 252-day simple moving average (SMA) and EWMA (FKO) methods dominate, particularly for larger values of N. Conversely, during the global financial crisis the MIDAS forecasts are those resulting in the smaller portfolio variance, especially when N is large. For small N during both high volatility periods, the DCC-CL method outperforms DECO however this is reversed as N increases. The dominance of equicorrelation in terms of the M-GARCH specifications is a trend seen previously for the entire out-of-sample period. It appears that in times of relative market stability as seen in the low volatility period, the simple methods result in superior forecasts in terms of smaller portfolio variances however these gains are of a smaller magnitude during this period than those seen during the higher volatility periods. As mentioned previously for the full out-of-sample, the assumption of equicorrelation appears to be useful in this context of portfolio allocation.

Table 5 also produces similar results to its full-sample counterpart, although the EWMA (FKO) method appears to perform better in the context of relatively low volatility than the other models. This suggests a regime switching type methodology may be worthwhile when targeting the variance of a portfolio when N is large, given the complex models are contained in the MCS for the periods of higher overall volatility. There is a difference between the two high volatility periods, with the GFC period resulting in much higher levels of volatility than those previously seen. For moderate N, the simple methods are superior to the M-GARCH models with the latter left out of the MCS. Differences between the simple methods themselves are difficult to establish during the GFC high-volatility period with MIDAS, SMA and EWMA (FKO) all included. This was not the case for the previous high-volatility period (corresponding to the 2001-2002 turbulence), where MIDAS and EWMA (FKO) form the MCS for the 10 asset portfolio and EWMA (FKO) the only model included in the MCS for the 50 asset portfolio. For large N, the MIDAS model is seen to be the superior model during the GFC, with all other models also included with the exception of DECO and the equally-weighted portfolio. This is in contrast to the previous high volatility period, which has a smaller MCS for large N and EWMA (FKO) is the model which dominates. Interestingly, the DECO model is included during this period for the 200 asset portfolio and DCC-CL removed. This suggests the usefulness of the assumption of equicorrelation may be limited in times of very high overall market volatility, however obtaining good forecasts using any method during these periods is difficult at best.

The focus now turns to more practical considerations of forecasting and portfolio allocation. Two issues are considered: firstly, the stability of the portfolios across time for each forecasting method, measured by the absolute weight changes for each portfolio; secondly, computation time is considered. The stability of the portfolios is considered to be a useful proxy for any economic value differences between the competing methods, without the need to make any assumptions regarding transaction costs. Obviously the equally-weighted portfolio is not included in this analysis. The absolute percentage weight change at time t for a given asset i, is calculated as $\boldsymbol{wc}_{i,t} = |(\boldsymbol{w}_{i,t} - \boldsymbol{w}_{i,t-1})/\boldsymbol{w}_{i,t-1}|$. Stability is measured by calculating the median absolute weight change for each asset in a portfolio, i, and taking the mean across the N assets: $\mu_{MED} = (\sum_{i=1}^{N}(\text{median}(\boldsymbol{wc}_{i,t})))/N$.

Results for the entire out-of-sample period are contained in Table 6. The M-GARCH methods, especially DCC-CL and its estimation-free equivalent DCC-EX, are the most stable for all N with μ_{MED} values ranging from 0.0521 to 0.0714 for both across the various portfolio sizes. The simple methods are comparatively much more volatile in terms of change in weights over the forecast period, for example the SMA values range from 0.0607 to 0.3235 across various portfolio sizes. Of the simple methods, the SMA provides more stable portfolio weights for the out-of-sample period. Similar results are obtained when taking into account periods of relatively high and low overall volatility (Table 7). From the economic point of view, the relative instability of the simple models may inhibit the gains seen in the context of portfolio volatility.

The computational burden of the more complex M-GARCH models compared to the simple models is shown in terms of CPU time taken to compute all 2029 correlation forecasts for the entire out-of-sample period. Individual times for each one-step-ahead forecast are not provided as they are too small to be of any use for the simple approaches and small portfolio sizes. CPU times are only provided for the estimation-free versions DCC and DECO, that is DCC-EX and DECO-EX to enable fair comparison with the simple approaches, none of which require any estimation of parameters. The similar performance of the daily estimation versions of the M-GARCH methods with their estimation-free counterparts in the volatility context provides further motivation for this. The underlying univariate GARCH estimation used to obtain the volatility-standardised returns series' is not included in this comparison. These times are included in Table 8. Table 9 provides only the time taken to estimate and re-estimate the parameters for DCC-CL and DECO on a daily basis. As an expanding window approach was used, the sample used to estimate the parameters for $t + 1$ extends by one observation each re-estimation and so the length of time taken for the entire out-of-sample period (ultimately resulting in 2029 forecasts) is provided. Overall, the M-GARCH methods take much longer to run, even without taking into account the daily re-estimation of the correlation parameters. This is especially true of the large N case. Practically speaking, in the context of large portfolios the aforementioned stability gains of implementing complex methodologies must be considered carefully given simpler methodologies provide forecasts of comparable (and often superior) quality for less computation time.

		EQ-W	MIDAS (0.98)	SMA	EWMA (FKO)	DCC-CL	DECO	DCC-EX	DECO-EX
Period	N	s	s	s	s	s	s	s	s
High									
2001:2477	5	22.5542	20.5623	20.6080	20.5498	20.7959	20.8135	20.7955	20.8114
	10	18.0249	16.5377	17.6398	16.4128	17.8018	18.1200	17.8049	18.0447
	50	21.5173	10.3105	10.9072	9.9592	15.7706	13.4033	15.7708	13.3330
	100	19.3495	9.7945	9.4793	9.4621	13.2108	11.6146	13.2108	11.6542
	200	17.7955	11.6215	9.6582	9.7552	12.7741	10.5026	12.7741	10.4879
3313:4029	5	25.9225	24.6223	24.7435	24.5521	24.7674	24.8465	24.7678	24.8350
	10	25.9542	19.9526	20.5037	19.8218	21.9781	21.4983	21.9787	21.5125
	50	28.3710	16.8243	16.4568	16.6012	23.8928	20.2936	23.8928	20.1767
	100	26.9356	18.3481	19.2258	18.6738	23.4713	22.9196	23.4712	23.0327
	200	27.0045	20.0901	20.1766	21.1875	23.6387	22.9919	23.6387	22.9345
Low									
2478:3312	5	18.0584	17.2780	17.0882	17.2521	17.0244	17.1006	17.0246	17.0902
	10	14.3145	12.5609	12.7645	12.4105	13.0180	12.9954	13.0182	12.9840
	50	12.1912	7.9533	8.5341	7.8177	10.2524	9.7236	10.2524	9.6531
	100	11.0929	6.5666	6.8731	6.1164	9.3333	8.4567	9.3333	8.4360
	200	10.6260	9.7141	9.2515	7.3815	9.2945	9.5601	9.2945	9.4838

Table 4: Out-of-sample summary statistics of minimum-variance portfolio returns for each volatility timing strategy, split into 'high' and 'low' volatility: s is the annualised percentage volatility of portfolio returns. In-sample period of 2000, entire period spanning 03/01/94 - 31/12/09.

Period	N	EQ-W	MIDAS (0.98)	SMA	EWMA (FKO)	DCC-CL	DECO	DCC-EX	DECO-EX
High									
2001:2477	5	0.0240	0.8980*	0.8980*	1.0000*	0.1450*	0.0930*	0.1450*	0.0930*
	10	0.0000	0.2280*	0.0010	1.0000*	0.0000	0.0000	0.0000	0.0000
	50	0.0000	0.0140	0.0070	1.0000*	0.0020	0.0020	0.0020	0.0020
	100	0.0010	0.0010	1.0000*	0.9910*	0.0010	0.0010	0.0010	0.0010
	200	0.0000	0.0000	1.0000*	0.7900*	0.0000	0.0580*	0.0000	0.0580*
3313:4029	5	0.0820*	0.4660*	0.4660*	1.0000*	0.4660*	0.0820*	0.4660*	0.0820*
	10	0.0420	0.1720*	0.0760*	1.0000*	0.0420	0.0420	0.0420	0.0420
	50	0.0050	0.1310*	1.0000*	0.7180*	0.0050	0.0050	0.0050	0.0050
	100	0.0010	1.0000*	0.4230*	0.5470*	0.2260*	0.0010	0.2260*	0.0010
	200	0.0410	1.0000*	0.9490*	0.8200*	0.3100*	0.0410	0.3100*	0.0410
Low									
2478:3312	5	0.0060	0.0190	0.2870*	0.0190	0.2870*	0.0190	1.0000*	0.0190
	10	0.0000	0.0070	0.0070	1.0000*	0.0070	0.0070	0.0070	0.0070
	50	0.0000	0.0170	0.0000	1.0000*	0.0000	0.0000	0.0000	0.0000
	100	0.0000	0.0040	0.0040	1.0000*	0.0000	0.0000	0.0000	0.0000
	200	0.0000	0.0000	0.0000	1.0000*	0.0000	0.0000	0.0000	0.0000

Table 5: Empirical MCS of out-of-sample global minimum-variance portfolio. Range MCS p-values are used; * indicates the model is included in the MCS with 95% confidence.

N	MIDAS (0.98)	SMA	EWMA (FKO)	DCC-CL	DECO	DCC-EX	DECO-EX
	μ_{MED}	μ_{MED}	μ_{MED}	μ_{MED}	μ_{MED}	μ_{MED}	μ_{MED}
5	0.0697	0.0607	0.0707	0.0581	0.0623	0.0580	0.0623
10	0.0956	0.0721	0.0963	0.0531	0.0903	0.0532	0.0831
50	0.1597	0.1374	0.1558	0.0521	0.1210	0.0521	0.1198
100	0.2042	0.1793	0.1944	0.0601	0.1609	0.0601	0.1598
200	0.3680	0.3235	0.3241	0.0714	0.2064	0.0714	0.2036

Table 6: Mean, μ_{MED}, of the Median (MED) of the absolute change in portfolio weights across each model for the out-of-sample period. In-sample period of 2000, entire period spanning 03/01/94 - 31/12/09.

Period	N	MIDAS (0.98)	SMA	EWMA (FKO)	DCC-CL	DECO	DCC-EX	DECO-EX
		μ_{MED}	μ_{MED}	μ_{MED}	μ_{MED}	μ_{MED}	μ_{MED}	μ_{MED}
High								
2001:2477	5	0.0940	0.0680	0.0893	0.0651	0.0689	0.0650	0.0689
	10	0.1156	0.0893	0.1131	0.0725	0.1040	0.0726	0.0993
	50	0.1747	0.1346	0.1788	0.0624	0.1316	0.0624	0.1323
	100	0.2912	0.2367	0.2864	0.0928	0.2456	0.0928	0.2467
	200	0.3966	0.3621	0.3695	0.0949	0.2436	0.0949	0.2428
3313:4029	5	0.0703	0.0697	0.0723	0.0629	0.0670	0.0629	0.0667
	10	0.1067	0.0736	0.1069	0.0547	0.1035	0.0547	0.0937
	50	0.2087	0.1760	0.2029	0.0668	0.1781	0.0668	0.1735
	100	0.2163	0.1999	0.2089	0.0673	0.1818	0.0673	0.1801
	200	0.3153	0.2979	0.2819	0.0721	0.1959	0.0721	0.1930
Low								
2478:3312	5	0.0648	0.0556	0.0656	0.0536	0.0580	0.0536	0.0581
	10	0.0836	0.0733	0.0861	0.0472	0.0814	0.0472	0.0736
	50	0.1320	0.1297	0.1206	0.0430	0.1050	0.0430	0.1042
	100	0.1706	0.1535	0.1607	0.0493	0.1300	0.0493	0.1293
	200	0.4162	0.3402	0.3599	0.0676	0.2081	0.0676	0.2053

Table 7: Mean, μ_{MED}, of the Median (MED) of the absolute change in portfolio weights across each model, split into periods of ‘high’ and ‘low’ volatility. In-sample period of 2000, entire period spanning 03/01/94 - 31/12/09.

N	MIDAS (0.98)	SMA	EWMA (FKO)	DCC-EX	DECO-EX
5	00:00:04	00:00:02	00:00:43	00:03:09	00:13:29
10	00:00:08	00:00:04	00:00:46	00:04:16	00:14:08
50	00:16:26	00:01:08	00:07:39	00:35:52	00:44:40
100	00:23:33	00:22:13	00:33:33	02:20:41	01:41:23
200	01:11:35	00:19:57	01:16:49	13:18:17	08:32:36

Table 8: CPU time of models (Hours : Minutes : Seconds), forecasting only. Entire period spanning 03/01/94 - 31/12/09 (2029 forecasts). Computer specifications: 12 core X5650 2.66GHz 64bit Intel Xeon processor.

N	DCC-CL	DECO
5	01:54:56	08:05:02
10	02:30:03	07:09:55
50	30:25:57	21:16:40
100	70:53:06	67:31:50
200	287:01:55	167:00:32

Table 9: CPU time of DCC-CL and DECO (Hours : Minutes : Seconds), estimation only. In-sample period of 2000 observations, entire period spanning 03/01/94 - 31/12/09. Computer specifications: 12 core X5650 2.66GHz 64bit Intel Xeon processor.

5 Conclusion

This paper provides a straightforward and commonsense approach toward investigating whether simpler moving average based correlation forecasting methods have equal predictive accuracy relative to their more complex multivariate GARCH counterparts for large dimensional problems. In an empirical setting, the advantages and disadvantages of popular ‘simple’ multivariate correlation forecasting methods (focusing on various moving average-type schemes) in comparison to more complex multivariate GARCH models, namely composite likelihood DCC (DCC-CL) and DECO are discussed. We find simple methods do achieve equal (and indeed superior) correlation forecasts in the context of minimising portfolio variance. The volatility of global minimum variance portfolios are used to compare methods from a volatility point of view, whilst portfolio weight stability and computation time are also considered. Comparisons of forecasting techniques are provided for the full out-of-sample period along with periods of higher and lower relative volatility, including the recent global financial crisis.

Overall, the EWMA (FKO) model performed best in terms of minimising the volatility of the forecasted portfolio. This superior performance was present for both high and low volatility periods, although the gains of using such a model were more significant for the low volatility period. The size of the Model Confidence Set increases as dimensionality does, however the 'simple' methods are consistently included in the set, for the full-sample, and high and low periods. This poses an interesting question of the suitability of a dynamic forecast combination of models. The exception to this was the MIDAS model, which is only included for higher values of N for the GFC crisis period. The relative instability of such methods however in terms of portfolio rebalancing may inhibit this superior predictive ability. These results provide insight for practitioners that so-called 'simple' methodologies are not inferior to more complex models in terms of covariance forecasting in larger dimensional problems.

In terms of the multivariate GARCH models used in this study, namely DCC-CL and DECO, the equicorrelation model outperforms the DCC-CL for moderate to large portfolios. This is thought to be due to the estimation error included in the necessary estimation of the unconditional correlation matrix for the DCC-CL and provides validity for the assumption of equicorrelation. Practically, the DCC-CL method achieved a more stable portfolio than the other methods (followed by DECO) and implementation of such a model may result in lower transaction costs. Of course, implementation of the M-GARCH forecasts does come at a cost in terms of computation time and this is found to be substantial as portfolio size increases. For practitioners there is an economic trade-off between the stability of the DCC-type model and the computational burden of using such methods.

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A List of Stocks

Ticker	Company Name	Sector	Ticker	Company Name	Sector
AA	Alcoa Inc.	Materials	BRE	BRE Properties Inc.	Financials
AAN	Aaron's Inc.	Consumer Discretionary	BRS	Bristow Group Inc.	Energy
AAON	AAON Inc.	Industrials	BSX	Boston Scientific Corp.	Health Care
AAPL	Apple Inc.	Information Technology	BW	Brush Engineered Materials	Materials
ABAX	Abaxis Inc.	Health Care	BWA	Borgwarner Inc.	Consumer Discretionary
ABFS	Arkansas Best Corp.	Industrials	BWS	Brown Shoe Company Inc.	Consumer Discretionary
ABT	Abbott Laboratories	Health Care	BXS	Bancorpsouth Inc.	Financials
ABX	Barrick Gold Corp.	Materials	BYD	Boyd Gaming Corp.	Consumer Discretionary
ACAT	Arctic Cat Inc.	Consumer Discretionary	BYI	Bally Technologies Inc.	Consumer Discretionary
ACE	ACE Ltd	Financials	CACI	CACI International Inc. (Cl A)	Information Technology
ACTL	Actel Corp.	Information Technology	CAG	Conagra Foods Inc.	Consumer Staples
ACV	Alberto-Culver Co.	Consumer Staples	CAH	Cardinal Health Inc.	Health Care
ADBE	Adobe Systems Inc.	Information Technology	CAKE	The Cheesecake Factory Inc.	Consumer Discretionary
ADCT	ADC Telecommunications Inc.	Information Technology	CAS	Castle (A.M.) & Co.	Materials
ADI	Analog Devices Inc.	Information Technology	CASY	Casey's General Stores Inc.	Consumer Staples
ADM	Archer-Daniels-Midland Co.	Consumer Staples	CAT	Caterpillar Inc.	Industrials
ADP	Automatic Data Processing	Information Technology	CATO	Cato Corp. (Cl A)	Consumer Discretionary
ADPT	Steel Excel Inc.	Information Technology	CB	Chubb Corp.	Financials
ADSK	Autodesk Inc.	Information Technology	CBB	Cincinnati Bell Inc.	Telecommunication Services
AEP	American Electric Power	Utilities	CBE	Cooper Industries PLC	Industrials
AES	AES Corp.	Utilities	CBK	Christopher & Banks Corp.	Consumer Discretionary
AF	Astoria Financial Corp.	Financials	CBR	Ciber Inc.	Information Technology
AFG	American Financial Inc.	Financials	CBRL	Cracker Barrel Old Country	Consumer Discretionary
AGCO	AGCO Corp.	Industrials	CBSH	Commerce Bancshares Inc.	Financials
AGL	AGL Resources Inc.	Utilities	CBT	Cabot Corp.	Materials
AGN	Allergan Inc.	Health Care	CCC	Calgon Carbon Corp.	Materials
AGYS	Agilysys Inc.	Information Technology	CCE	Coca-Cola Enterprises	Consumer Staples
AIN	Albany Intl Corp. (Cl A)	Industrials	CCK	Crown Holdings Inc.	Materials

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Ticker	Company Name	Sector	Ticker	Company Name	Sector
AIR	AAR Corp.	Industrials	CEC	CEC Entertainment Inc.	Consumer Discretionary
AIRM	Air Methods Corp.	Health Care	CEG	Constellation Energy Group	Utilities
AIT	Applied Industrial Tech Inc.	Industrials	CELG	Celgene Corp.	Health Care
AKR	Acadia Realty Trust	Financials	CFR	Cullen/Frost Bankers Inc.	Financials
ALB	Albemarle Corp.	Materials	CGNX	Cognex Corp.	Information Technology
ALE	Allete Inc.	Utilities	CHD	Church & Dwight Co Inc.	Consumer Staples
ALEX	Alexander & Baldwin Inc.	Industrials	CHG	CH Energy Group Inc.	Utilities
ALK	Alaska Air Group Inc.	Industrials	CHK	Chesapeake Energy Corp.	Energy
ALL	Allstate Corp.	Financials	CHS	Chico's Fas Inc.	Consumer Discretionary
ALOG	Analogic Corp.	Health Care	CHUX	O'Charleys Inc.	Consumer Discretionary
ALTR	Altera Corp.	Information Technology	CI	Cigna Corp.	Health Care
AMAT	Applied Materials Inc.	Information Technology	CINF	Cincinnati Financial Corp.	Financials
AMD	Advanced Micro Devices	Information Technology	CKP	Checkpoint Systems Inc.	Information Technology
AMGN	Amgen Inc.	Health Care	CKR	CKE Restaurants Inc.	Consumer Discretionary
AMR	AMR Corp.	Industrials	CL	Colgate-Palmolive Co.	Consumer Staples
ANN	ANN Inc.	Consumer Discretionary	CLC	Clarcor Inc.	Industrials
APA	Apache Corp.	Energy	CLP	Colonial Properties Trust	Financials
APC	Anadarko Petroleum Corp.	Energy	CLX	Clorox Company	Consumer Staples
APD	Air Products & Chemicals Inc.	Materials	CMA	Comerica Inc.	Financials
APOG	Apogee Enterprises Inc.	Industrials	CMC	Commercial Metals Co.	Materials
ARB	Arbitron inc.	Consumer Discretionary	CMCSA	Comcast Corp. (Cl A)	Consumer Discretionary
ARG	Airgas Inc.	Materials	CMI	Cummins Inc.	Industrials
ARRS	Arris Group Inc.	Information Technology	CMTL	Comtech Telecommunications	Information Technology
ARW	Arrow Electronics Inc.	Information Technology	CMVT	Comverse Technology Inc.	Information Technology
ASBC	Associated Banc-Corp.	Financials	CNL	Cleco Corporation	Utilities
ASGN	On Assignment Inc.	Industrials	CNP	Centerpoint Energy Inc.	Utilities
ASH	Ashland Inc.	Materials	CNW	Con-Way Inc.	Industrials
ATK	Alliant Techsystems Inc.	Industrials	COG	Cabot Oil & Gas Corp.	Energy
ATMI	ATMI Inc.	Information Technology	COHU	Cohu Inc.	Information Technology

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Ticker	Company Name	Sector	Ticker	Company Name	Sector
ATML	Atmel Corp.	Information Technology	COLB	Columbia Banking System Inc.	Financials
ATO	Atmos Energy Corp.	Utilities	COMS	3Com Corp.	Information Technology
ATR	Aptargroup Inc.	Materials	COO	The Cooper Cos Inc.	Health Care
AVA	Avista Corp.	Utilities	COP	Conocophillips	Energy
AVB	Avalonbay Communities Inc.	Financials	COST	Costco Wholesale Corp.	Consumer Staples
AVID	Avid Technology Inc.	Information Technology	CPB	Campbell Soup Co.	Consumer Staples
AVT	Avnet Inc.	Information Technology	CPRT	Copart Inc.	Industrials
AWR	American States Water Co.	Utilities	CPT	Camden Property Trust	Financials
AXE	Anixter International inc.	Information Technology	CPWR	Compuware Corp.	Information Technology
AYE	Allegheny Energy Inc.	Utilities	CR	Crane Co.	Industrials
AZO	Autozone Inc.	Consumer Discretionary	CREE	Cree Inc.	Information Technology
B	Barnes Group Inc.	Industrials	CRK	Comstock Resources Inc.	Energy
BA	The Boeing Co.	Industrials	CRS	Carpenter Technology	materials
BAC	Bank of America Corp.	Financials	CRY	Cryolife Inc.	Health Care
BBBY	Bed Bath & Beyond Inc.	Consumer Discretionary	CSC	Computer Sciences Corp.	Information Technology
BBT	BB&T Corp.	Financials	CSCO	Cisco Systems Inc.	Information Technology
BBY	Best Buy Co Inc.	Consumer Discretionary	CSH	Cash America Intl Inc.	Financials
BC	Brunswick Corp.	Consumer Discretionary	CSL	Carlisle Cos Inc.	Industrials
BCO	The Brink's Co.	Industrials	CSX	CSX Corp.	Industrials
BDX	Becton Dickinson & Co.	Health Care	CTAS	Cintas Corp.	Industrials
BEAV	BE Aerospace Inc.	Industrials	CTB	Cooper Tyre & Rubber	Consumer Discretionary
BEN	Franklin Resources Inc.	Financials	CTL	Centurylink Inc.	Telecommunication Services
BEZ	Baldor Electric	Industrials	CUB	Cubic Corp.	Industrials
BF/B	Brown-Foreman Corp. (Cl B)	Consumer Staples	CUZ	Cousins Properties Inc.	Financials
BGG	Briggs & Stratton	Industrials	CV	Central Vermont Public Serv.	Utilities
BHE	Benchmark Electronics Inc.	Information Technology	CVS	CVS Caremark Corp.	Consumer Staples
BHI	Baker Hughes Inc.	Energy	CVX	Chewron Corp.	Energy
BID	Sotheby's	Consumer Discretionary	CY	Cypress Semiconductor Corp.	Information Technology
BIG	Big Lots Inc.	Consumer Discretionary	CYN	City National Corp.	Financials

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Ticker	Company Name	Sector	Ticker	Company Name	Sector
BIIB	Biogen Idec Inc.	Health Care	CYT	Cytec Industries Inc.	Materials
BJS	BJ Services Co.	Energy	D	Dominion Resources Inc/VA	Utilities
BK	Bank of New York Mellon Corp.	Financials	DBRN	Dress Barn Inc.	Consumer Discretionary
BKE	The Buckle Inc.	Consumer Discretionary	DCI	Donaldson Co Inc.	Industrials
BKH	Black Hills Corp.	Utilities	DD	DU Pont (E.I.) de Nemours	Materials
BKS	Barnes & Noble Inc.	Consumer Discretionary	DDR	DDR Corp.	Financials
BLL	Ball Corp.	Materials	DDS	Dillards Inc. (Cl A)	Consumer Discretionary
BLUD	Immucor Inc.	Health Care	DE	Deere & Co.	Industrials
BMC	BMC Software Inc.	Information Technology	DELL	Dell Inc.	Information Technology
BMS	Bemis Co.	Materials	DFG	Delphi Financial Group (Cl A)	Financials
BMY	Bristol-Myers Squibb Co.	Health Care	DGII	Digi International Inc.	Information Technology
BNE	Bowne & Co Inc.	Information Technology	DHI	DR Horton Inc.	Consumer Discretionary
BOBE	Bob Evans Farms	Consumer Discretionary	DHR	Danaher Corp.	Industrials
BOH	Bank of Hawaii Corp.	Financials	DIN	Dineequity Inc.	Consumer Discretionary

Table 10: The 200 stocks included in the full dataset, including ticker code, company name and sector.